INTERPRETING THE KUIPER BELT LUMINOSITY FUNCTION. J. M. Hahn, *Lunar and Planetary Institute, Houston TX 77058-1113, USA, (hahn@lpi.jsc.nasa.gov)*, L. Brown, *Colgate University, Hamilton NY 13346, USA, (labrown@center.colgate.edu)*.

The Kuiper Belt luminosity function $\Sigma(m)$, which is the number density of Kuiper Belt Objects (KBOs) observable on the sky-plane at various limiting magnitudes m, is analyzed in order to assess whether the KBO radial surface density σ is $primordial$, meaning that σ decreases with heliocentric distance, or $eroded$ such that σ increases with distance. Although most treatments of the Kuiper Belt assume a primordial radial distribution, dynamical models suggest that the the Kuiper Belt has been eroded by the giant-planets' perturbations from the inside-out. It is shown below that if the Kuiper Belt is indeed eroded, then the size of the largest KBO must decrease with distance in order to agree with the observed dearth of large, distant KBOs. If this is so, then an eroded Kuiper Belt might be ~ 3 times more populous than previously thought.

We begin by assuming that the KBO cumulative size distribution varies as the power-law $N(R) = (R/R_{min})^{-Q}$, which is the $fractional$ number of KBOs having radii $> R$ out of a total population N_t having radii between $R_{min} \leq R \leq R_{max}$. If these KBOs are distributed in a disk having surface number density $\sigma(r) \propto r^{-\beta}$ between the heliocentric distances $r_1 \leq r \leq r_2$, then $\sigma(r) = [(1-\beta/2)/(\gamma_2^{2-\beta}-1)]$ $(r/r_1)^{-\beta} N_t/\pi r_1^2$ where $\gamma_2 \equiv r_2/r_1$. Provided the KBOs have a characteristic inclination i, their volume number density is $n(r) \simeq \sigma/2r\sin i$. The number of KBOs in a differential volume element that subtends a solid angle ϕ^2 is thus $d\mathcal{N}(r) = nr^2\phi^2 dr$. If this volume element is observed to a limiting magnitude m, then $d\Sigma(m) = N(R_m)d\mathcal{N}/\phi^2$ is the surface number density of visible KBOs on the sky plane, where R_m is the smallest visible object at r. Noting that $m \simeq m_1 - 5\log(R_m/R_{min}) + 10\log(r/r_1)$ where m_1 is the magnitude of the smallest, closest KBO of radius R_{min} at $r = r_1$, the surface number density of visible KBOs along a line of sight (e.g., the luminosity function) is

$$\Sigma(m) = \int_{r_1}^{r_{max}(m)} d\Sigma = \frac{,\, N_t}{\Omega} 10^{Q(m-m_1)/5} \qquad (1)$$

where , $\equiv (1-\beta/2)(1-\gamma_x^{-2Q'})/Q'(\gamma_2^{2-\beta}-1)$, $Q' \equiv Q - 1 + \beta/2$, the solid angle subtended by the Kuiper Belt is $\Omega \equiv 4\pi\sin i$, the distance to the largest, most distant KBO visible at the limiting magnitude m is $r_{max}(m) = \sqrt{R_{max}/R_{min}}\,10^{(m-m_1)/10} r_1$, and the dimensionless viewing depth $\gamma_x \equiv r_{max}/r_1$.

The luminosity function is a useful quantity since its logarithmic slope, $d\log\Sigma/dm \simeq Q/5$, yields an estimate of the Kuiper Belt size distribution Q. Gladman *et al.* (1998) report an R-band luminosity function of $\Sigma(m) = 10^{0.76(m-23.4)}$ KBOs/deg^2, so the KBO size distribution is $Q \simeq 3.8$.

Upon adopting a plausible Kuiper Belt model, one can estimate the total number of KBOs from Σ. Assuming that the Kuiper Belt spans $r_1 = 30$ to $r_2 = 50$ AU with KBO sizes ranging from $R_{min} = 20$ km to $R_{max} = 370$ km (the observed range of sizes), then $m_1 \simeq m_\odot - 2.5\log[a(R_{min}/1\text{ AU})^2(r_1/1\text{ AU})^{-4}) = 25.3$ is the R-band magnitude of the smallest, closest KBO assuming an $a = 0.04$ albedo. If we consider an observation to a limiting magnitude of $m = 23.4$, then $\Sigma(m) = 1.0$ deg^{-2} is the expected sky-density of KBOs, and the largest Kuiper Belt is visible at the Belt's assumed outer edge so $r_{max} = 50$ AU. If one adopts a $primordial$ KBO surface density that decreases with distance as, say, $\beta \simeq 1.5$, then the preceding constants are $Q' = 3.55$, $\gamma_2 = \gamma_x = 1.67$, and , $\simeq 0.25$. The inclinations of KBOs is about $i \sim 15^\circ$ (Jewitt *et al.* 1996), so the Kuiper Belt subtends a solid angle of about $\Omega \sim 10^4$ deg^2. Inserting these quantities into Eq. (1) suggests that there are $N_t \sim 10^6$ KBO of radii $20 < R < 370$ km between $30 < r < 50$ AU.

However the assumption that the Kuiper Belt's surface density might be primordial, *i.e.*, that $\sigma(r)$ decreases with distance, is suspect. Dynamical models show that perturbations by Neptune tend to erode a primordial Kuiper Belt from the inside-out (see Fig. 8 of Duncan *et al.* 1995). These simulations show that the surface density of an eroded Belt instead grows rapidly with r which, if fit by a power-law, would vary anywhere between $\beta \sim -5$ to -10 out to Neptune's 2:1 mean-motion resonance at $r = 48$ AU. It should also be noted that these simulations considered erosion by planets in rather static orbits. If Neptune had instead experienced a history of orbital expansion (e.g., Malhotra 1995), the Kuiper Belt might be more severely eroded. Dynamical erosion also alters the above estimate of the KBO population. Assuming the current Kuiper Belt is quite eroded, say, with $\beta \simeq -8$, then , $\simeq 0.06$. Keeping all other Kuiper Belt parameters unchanged shows that the prior estimate of N_t could be underestimated by a factor of about 4, due to the fact that most KBOs in an eroded disk orbit unseen at greater heliocentric distances.

Next, consider whether the observed distribution of KBO distances is consistent with an eroded disk that is most populated at larger r. Dividing the Belt into a near zone between $r_1 < r < r_2$ and a far zone $r_2 < r < r_3$, Eq. (1) shows that the ratio of observable KBOs in the far/near zones is

$$f \equiv \frac{\Sigma(r_2 < r < r_3)}{\Sigma(r_1 < r < r_2)} = \frac{1 - \left(\frac{r3}{r2}\right)^{-2Q'}}{\left(\frac{r2}{r1}\right)^{2Q'} - 1} \qquad (2)$$

(cf. Gladman *et al.* 1998). First, consider a primordial disk with $\beta = 1.5$. For a near zone between $r_1 = 30$ AU and $r_2 = 45$ AU and a far zone out to $r_3 = 50$ AU, the relative fraction of far/near KBOs is $f \simeq 0.03$. This fraction is actually quite consistent with the current ratio of 3 of 88 KBOs having $r > 45$ AU. Note, however, that the eroded disk example with $\beta = -8$ is seemingly inconsistent with observations since the abundance of far/near KBOs would be $f \simeq 0.5$!

It should be noted that in a magnitude-limited survey, these distant and seemingly overabundant KBOs predicted by the eroded disk hypothesis are at the large end of the KBO size spectrum. Thus the eroded disk hypothesis can still be

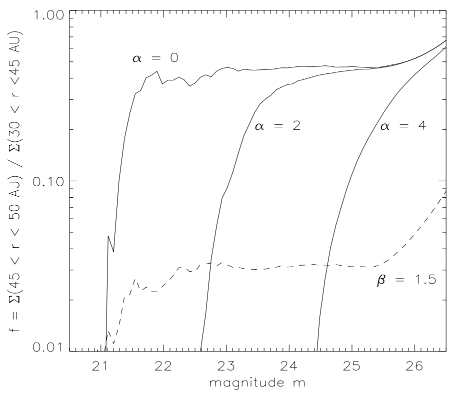

Figure 1: The ratio f of KBOs observed in the far and near zones versus limiting magnitude m. These curves are obtained from a Monte Carlo model composed of KBOs randomly distributed radially between $r_1 = 30$ AU and $r_3 = 50$ AU with a size distribution $Q = 3.8$ between $R_{min} = 20$ km and $R_{max}(r) = 370(r/r_1)^{-\alpha}$ km. The dashed curve is for a primordial disk having $\beta = 1.5$, and the solid curves are for an eroded $\beta = -8$ disk for selected values of α.

salvaged by removing the problematic KBOs by assuming that the radius of the largest KBO *decreases* with depth. This is in fact quite plausible since KBO accretion rates decrease with heliocentric distance so that one might expect smaller R_{max} at larger r. Upon assuming that R_{max} varies as a power law with distance, $R_{max} \propto (r/r_1)^{-\alpha}$, Fig. 1 shows how the far/near ratio varies with limiting magnitude m for various values of α. The fairly deep survey by Gladman *et al.* (1998) searched down to $m = 25.6$ and found zero of 5 KBOs lying in the far zone. This suggests an $f \lesssim 1/5$ and thus $\alpha \gtrsim 4$. This finding is a bit steeper than might be expected from ordered (e.g., non-runaway) accretion theory, which shows that the radius of a planetesimal grows at the rate $\dot{R} \propto \sigma\Omega$ when embedded in a disk of surface density σ having a mean motion Ω (Ward 1996). After time t_g, the largest planetesimal will have a radius $R_{max}(r) = \dot{R}t_g \propto r^{-\beta-3/2}t_g$. If t_g is the time when growth is later arrested, perhaps due to the sudden formation of Neptune (Stern and Colwell 1997) and/or the onset of radial migration and resonance sweeping by that planet (Malhotra 1995), then t_g is independent of r and $R_{max}(r) \propto r^{-3}$ while disk is initially in its primordial $\beta \sim 1.5$ state. If these suppositions are correct, then the size of the largest KBO in an eroded disk might shrink from $R_{max} \simeq 370$ km at 30 AU down to $R_{max} \simeq 50$ km at 50 AU.

A decrease in R_{max} with radial distance also tends to steepen the luminosity function, as shown in Fig. 2. The dashed curve is the observed Kuiper Belt luminosity function, while the narrow curve is for an eroded disk having a $Q = 3.8$ size distribution and $\alpha = 4$. However better agreement with observations is achieved with a shallower size distribution $Q = 2.8$, which implies a total KBO population of $N_t \sim 3 \times 10^6$ objects having radii $20 < R < 370$ km between $30 < r < 50$ AU.

If the Kuiper Belt is in fact eroded such that its surface density σ increases with distance as rapidly as dynamical models suggest, then several observational constraints are imposed upon the KBO size and radial distributions. The dearth of distant large KBOs indicates that the size of the largest KBO decreases with distance, perhaps as $R_{max} \propto r^{-4}$ (or faster). This is consistent with KBOs having formed via orderly growth in a disk having $\sigma \propto r^{-\beta}$ with an initial $\beta \sim 2.5$ or more. Also, KBO growth must have been terminated suddenly throughout the disk, presumably due to stirring by Neptune. Perturbations by this planet would subsequently erode the disk from the inside out such that $\beta \to$ negative. Since most of the surviving KBOs would be of small size and reside at larger heliocentric distances, astronomers might experience more rapid KBO discoveries by observing the Kuiper Belt to a greater depth rather than via shallow wide-angle surveys. Confirmation of the eroded disk hypothesis could be achieved with a deep survey ($m > 26$ in R) that detects KBOs in the far ($45 < r < 50$ AU) and near ($30 < r < 45$ AU) zones at a relative abundance as high as $f \sim 0.5$. If the Kuiper Belt is indeed eroded, its estimated population may be ~ 3 times larger than previously thought.

References: Duncan, Levison, and Budd, 1995, *AJ*, **110**, 3073. Gladman, Kavelaars, Nicholson, Loredo, and Burns, 1998, *AJ*, **116**, 2042. Jewitt, Luu, and Chen, 1996, *AJ*, **112**, 1225. Malhotra, 1995, *AJ*, **110**, 420. Stern and Colwell, 1997, *AJ*, **114**, 841. Ward, 1996, *ASP Conf. Series*, **107**, 337.

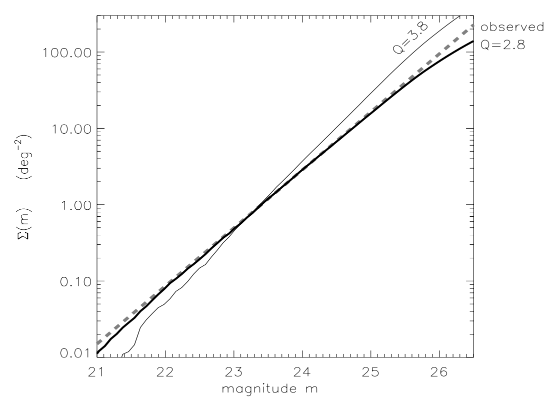

Figure 2: The dashed curve is the observed Kuiper Belt luminosity function Σ versus magnitude m. The steeper, narrow curve is obtained from a Monte Carlo model of an eroded $\beta = -8$ disk composed of $N_t = 9 \times 10^6$ KBOs having a size distribution $Q = 3.8$ and $\alpha = 4$. The darker curve shows that better agreement with observations is achieved with a shallower $Q = 2.8$, $\alpha = 4$ Kuiper Belt having $N_t = 3 \times 10^6$ KBOs.