Simulations of the B ring’s outer edge

The B ring’s outer edge is controlled by an $m = 2$ inner Lindblad resonance with the satellite Mimas, with the ring-edge lying 14 km exterior to this resonance (Spitale & Porco 2010). As expected, the B ring-edge has an $m=2$ component with an epicyclic amplitude of $R = 35$ km, which trails Mimas by $\phi = -5^\circ$ (Spitale & Porco 2010). Cassini observations also show that the ring-edge has $m = 1$ and $m = 3$ disturbances there, as well as a freely precessing $m = 2$ component, but these disturbances are not yet accounted for by this model, and are not considered further here.

Drag-dominated ring: Figure 1 shows a simulated B ring after it settles to equilibrium at time $t = 50$ yrs or $4 \times 10^7$ orbits. Dots indicate particle positions, and curves connect particles along streamlines. A compressible equation of state is used here, and dissipation due to a fictitious drag having $C_D = 3 \times 10^{-3}$ causes the ring’s $m = 2$ pattern to lap behind Mimas by $\phi = -5^\circ$. A ring having a higher surface density will have a smaller epicyclic amplitude $\kappa$, and this simulated ring’s surface density $\sigma_0 = 100$ gm/cm$^2$ was chosen to match the B ring’s observed amplitude $\kappa = 35$ km.

Vorticity-dominated ring: Figure 2 shows a more realistic simulation where dissipation in the ring is due to viscous friction that accounts for other unmodelled effects, such as collisions or scattering among particles. A rather heavy viscosity, $\nu_c = 600$ cm$^2$/sec and $\nu_t = 5 \times 10^{-1}$, is used to make the ring’s $m = 2$ pattern lag behind Mimas’ longitude by $\phi = -5^\circ$. Note also the banded pattern of the ring’s surface density, which is due to alternating separations among adjacent streamlines. This banding is only seen when dissipation in the ring is dominated by viscosity, which suggests that this effect might be associated with viscous overstability. Note also that this banding is most prominent at the ring’s outer edge, which is most strongly disturbed by Mimas $m = 2$ ILR. The bands’ radial spacing is $\Delta r \sim 10$km in these simulations, but this only an upper limit because ongoing higher-resolution simulations show similar banding on smaller scales.

Figure 3 shows the ring just before then. Surface density variations also cause variations in the ring’s vertical thickness $h = \sigma/2p$, shown as radial cuts along the ring’s longitude of periapse and apoaopse.

Note that the outcome seen in Fig. 3 might be a consequence of boundary conditions, which treats the simulated ring’s inner and outer streamlines as hard edges. Consequently, spiral waves do not escape the simulation, which may be why the ring does not settle to equilibrium. This will be examined closely as we implement a radiative boundary condition that allows inward-propagating waves to escape. Lastly, note that this model does not yet account for the ring’s vertical gravity, which will increase ring pressure and, we suspect, cause spiral waves to propagate even faster.

Spiral density waves

Figure 4 demonstrates that the code developed here can also simulate nonlinear spiral density waves launched at a satellite’s m Lindblad resonances. This simulation of an $m = 2$ spiral used 1.5k particles, and was evolved for 5000 orbits, which is the time for this wave to propagate about six wavelengths. Execution time on a desktop PC is 30 minutes.

Main findings

- An Nbody method is successfully used to simulate collective phenomena in a planetary ring, such as the scalling that occurs along the outer edge of Saturn’s B ring, and the propagation of a spiral density wave in the ring’s interior.
- The code uses streamline dynamics to calculate the ring’s internal forces: gravity, pressure, and viscosity. This method is very effective at mitigating the particle-particle scattering that often prevents an Nbody code from resolving collective effects in a ring simulation.
- The purpose of the effort is to compare simulations to Cassini observations of the ring, to infer the ring’s physical properties (surface density, equation of state, etc), and to test theories of ring evolution (such as the origin of the B ring’s interesting $m = 2$ modes).
- The next generation of this code will also track the ring’s vertical displacements, which will then allow us to simulate the propagation of non-linear spiral bending waves as well.

References